Systèmes entrée-sortie non linéaires et applications en audio-acoustique

Séries de Volterra

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Ecole Thématique ”Théorie du Contrôle en Mécanique”
2019

Plan

1. Préambule
2. Séries de Volterra : généralités
3. Calcul des noyaux de Volterra d'un système différentiel
4. Exercices et applications en audio-acoustique
5. Convergence
6. Extension en dimension infinie et application
7. Conclusion

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- Lois d'interconnexion *(somme, produit, cascade)*
- Calcul algébrique de noyaux de transfert
- Réalisation numérique

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Computing $y(t) = y_a(t) + y_b(t)$

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\begin{aligned}
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&\quad + \sum_{n=1}^{+\infty} \int_{\mathbb{R}^n} b_n(\tau_{1:n}) u(t-\tau_1) \ldots u(t-\tau_n)\, \mathrm{d}\tau_1 \ldots \mathrm{d}\tau_n
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Result: Equivalent kernels c_n

$$c_n(\tau_{1:n}) = a_n(\tau_{1:n}) + b_n(\tau_{1:n})$$

Laplace T.: $$C_n(s_{1:n}) = A_n(s_{1:n}) + B_n(s_{1:n})$$

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$$\times \sum_{q=1}^{+\infty} \int_{\mathbb{R}^q} b_q(\sigma_{1:q}) u(t-\sigma_1)\dots u(t-\sigma_q)\, d\sigma_1 \dots d\sigma_q$$

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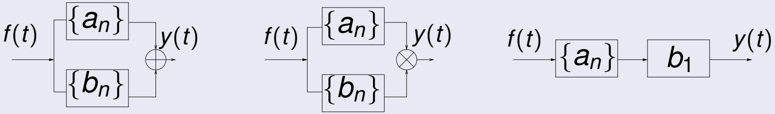
Interconnection laws: Sum, Product and Cascade



Equivalent transfer kernels C_n

where $\mathbb{M}_n^m = \{(p_1, \ldots, p_m) \in (\mathbb{N}^*)^m$ s.t. $p_1 + \cdots + p_m = n\}$

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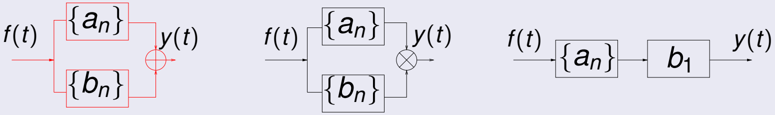

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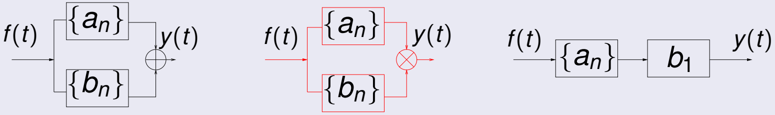

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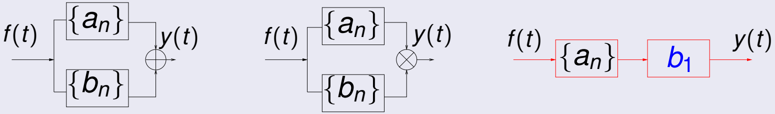

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(b_1: linear) with $\widehat{s_{1:n}} = s_1 + \ldots + s_n$

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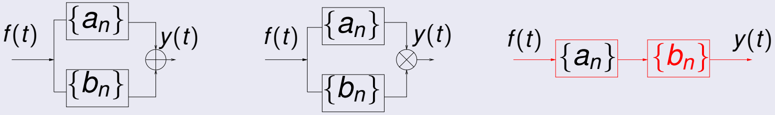

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Cascade: $C_n(s_{1:n}) = \sum_{m=1}^{n} \sum_{p \in \mathbb{M}_n^m} A_{p_1}(s_{1:p_1}) \ldots A_{p_m}(s_{p_1+\ldots+p_{m-1}+1:n})$

(general case) $.\, B_m\big(\widehat{s_{1:p_1}}, \ldots\ldots, \widehat{s_{p_1+\ldots+p_{m-1}+1:n}}\big)$

where $\mathbb{M}_n^m = \{(p_1, \ldots, p_m) \in (\mathbb{N}^*)^m \text{ s.t. } p_1 + \cdots + p_m = n\}$

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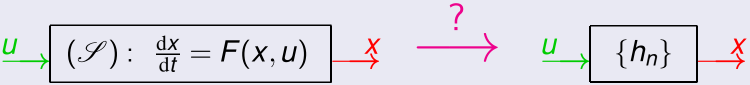
How to derive the Volterra kernels of a system $\mathscr{S}$?

Goal: Find the Volterra kernels $\{h_n\}$ of $(\mathscr{S})$ where



Several methods are available... [Brockett,Isidori,Rugh,Boyd]

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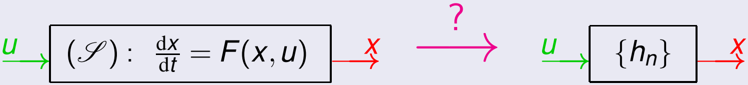

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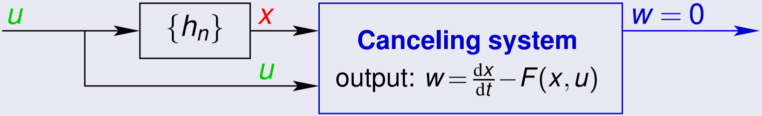
<u>Here:</u> Introduce the "canceling system" of $\mathscr{S}$



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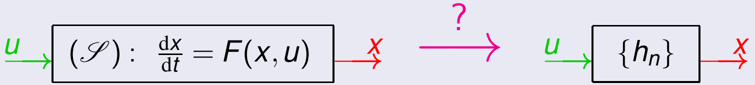

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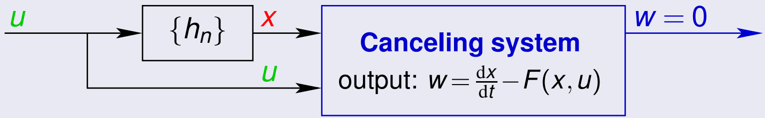

Principle: this cascade defines the **null system** $u \rightarrow \{z_n = 0\} \rightarrow w$

Interconnection laws gives the equations satisfied by $\{h_n\}$.

Example: nonlinear spring

$$\xrightarrow{f} \boxed{\{h_n\}} \xrightarrow{z}$$

Equation (at rest before t=0)

$$mz'' + az' + k_1 z + k_2 [z]^2 = f$$

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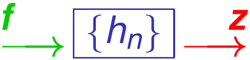

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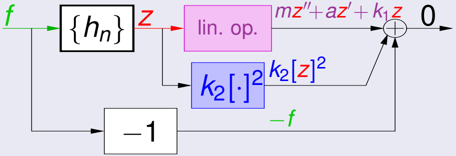

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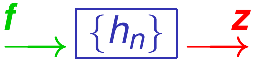

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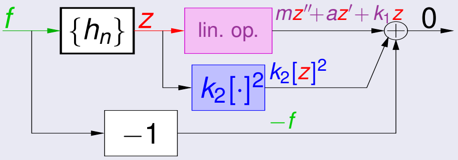

Elementary blocks → equivalent transfer kernels

$m\frac{\mathrm{d}^2}{\mathrm{d}t^2} + a\frac{\mathrm{d}}{\mathrm{d}t} + k_1$	$\rightarrow$	$Q_1(s) = ms^2 + as + k_1$, $Q_n = 0$ si $n \geq 2$
$k_2[\cdot]^2$	$\rightarrow$	interconnection "product" and $\times k_2$
-1	$\rightarrow$	$-\delta_{1,n} = -1$ if $n = 1$ and $-\delta_{1,n} = 0$ otherwise

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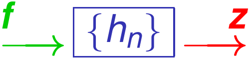

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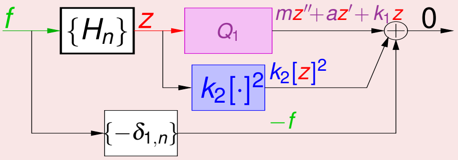

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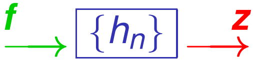

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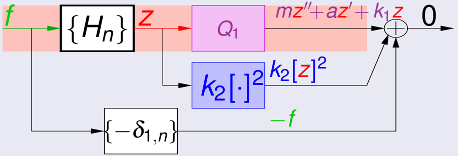

Kernel of order *n* of the *canceling system*

$$H_n(s_{1:n})Q_1(\widehat{s_{1:n}})$$

$$+ \quad k_2 \sum_{p=1}^{n-1} H_p(s_{1:p})H_{n-p}(s_{p+1:n})$$

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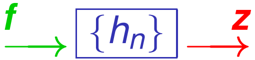

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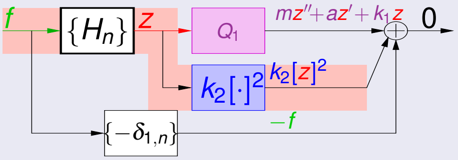

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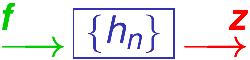

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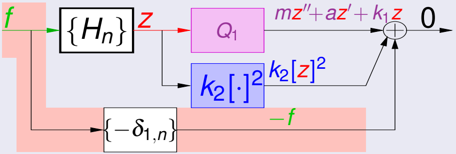

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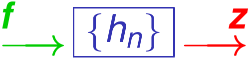

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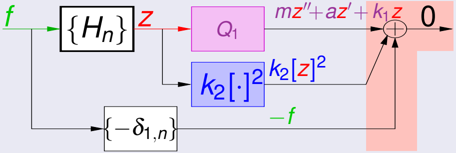

Kernel of order *n* of the *canceling system*

$$\begin{aligned} & H_n(s_{1:n})Q_1(\widehat{s_{1:n}}) \\ + \quad & k_2 \sum_{p=1}^{n-1} H_p(s_{1:p})H_{n-p}(s_{p+1:n}) \\ + \quad & -\delta_{1,n} \qquad = 0 \end{aligned}$$

$\longrightarrow$ linear eq. w.r.t. H_n.

Kernels $\{H_n\}$ of the system $\xrightarrow{f} \boxed{\{h_n\}} \xrightarrow{z}$

General solution: recursive algebraic equation ($n \geq 1$)

$$H_n(s_{1:n}) = \frac{\delta_{1,n} - k_2 \overbrace{\sum_{p=1}^{n-1} H_p(s_{1:p}) H_{n-p}(s_{p+1:n})}^{\text{orders } < n}}{Q_1(\widehat{s_{1:n}})}$$

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First transfer kernels ($n = 1, 2, 3$,etc)

$H_1(s_1) = 1/Q_1(s_1)$, (second order AR filter)

$H_2(s_{1:2}) = -k_2\, H_1(s_1)\, H_1(s_2)\, H_1(\widehat{s_{1:2}})$,

$H_3(s_{1:3}) = -k_2 \left[H_2(s_{1:2})\, H_1(s_3) + H_1(s_1)\, H_2(s_{2:3})\right] H_1(\widehat{s_{1:3}})$,

etc.

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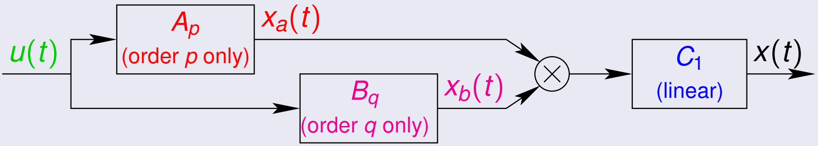
Consider the following **“elementary system”**



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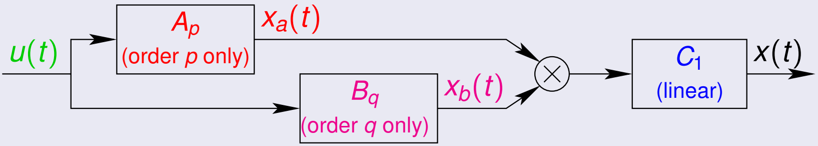
Consider the following "elementary system"



Equivalent system (using "product" & "cascade")

The Volterra transfer kernels of this system are all zero except

$$H_{p+q}(s_{1:p+q}) = A_p(s_{1:p})\, B_q(s_{p+1:p+q})\, C_1(\widehat{s_{1:p+q}})$$

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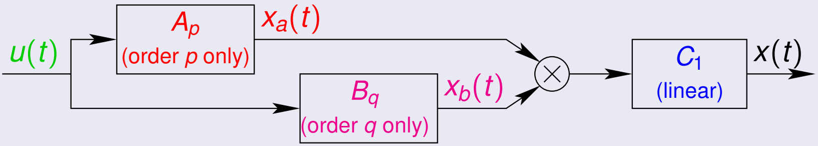

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Method:

Recursively build a realization as a **sum of such systems**.

Example: nonlinear spring

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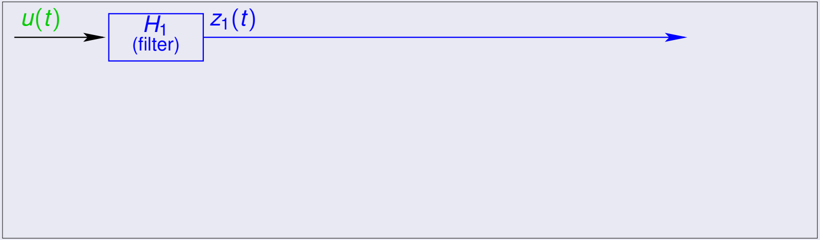

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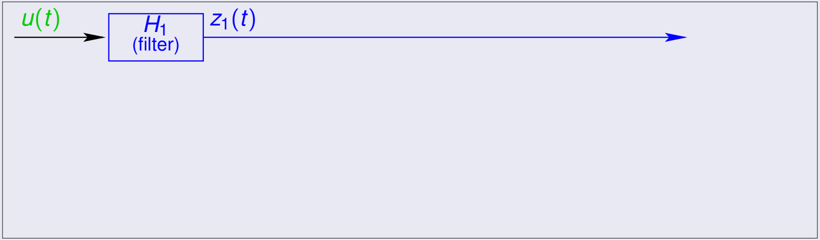

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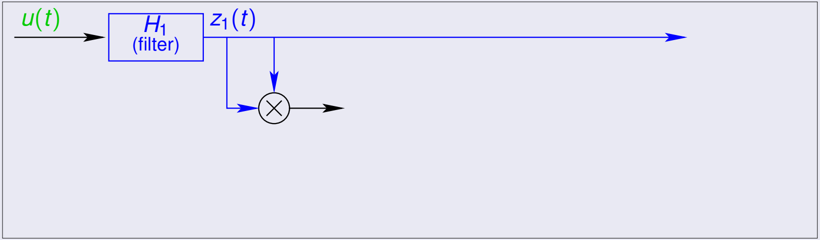

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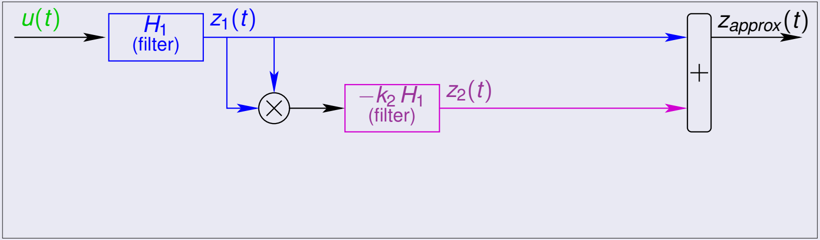

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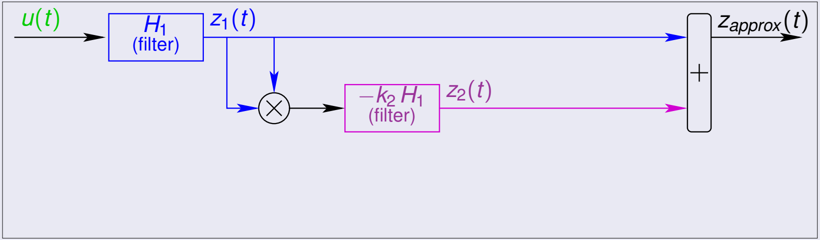

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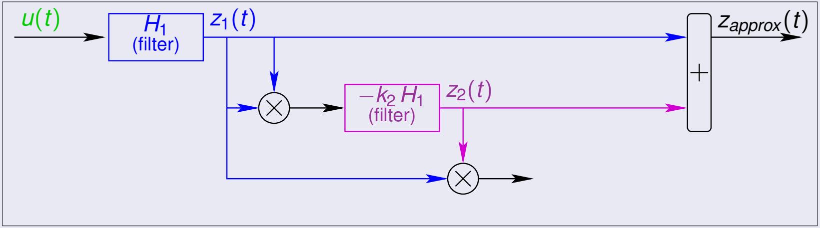

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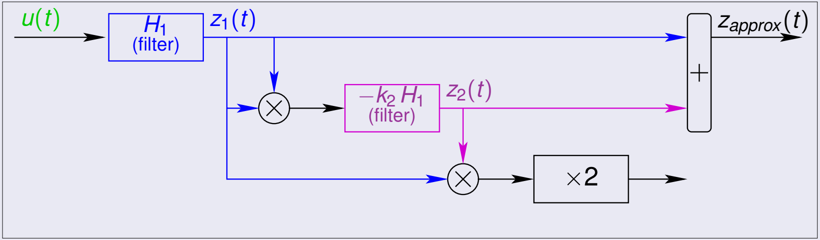

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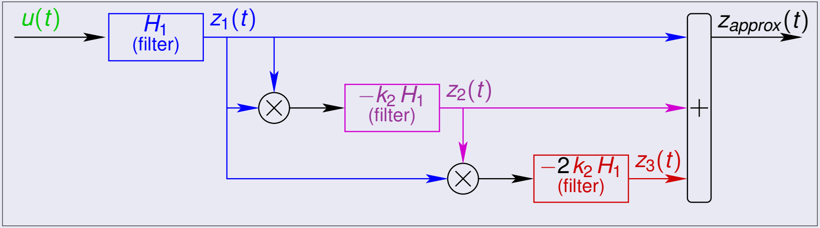

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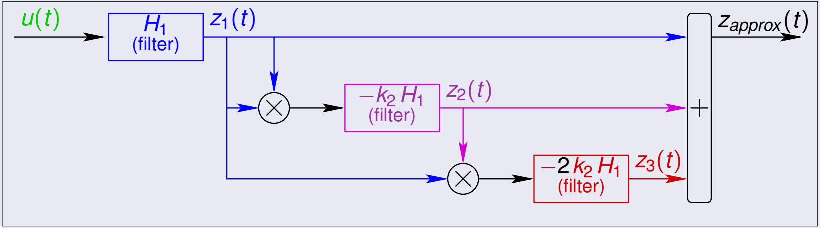

RESULT: The system is composed of **sums**, **products** and

linear systems (filters) ($\longrightarrow$ standard digital versions!)

Rk: if linear systems are stable, the system is stable!

Aliasing rejection in simulations

Product of signals

signal	$a(t)$	$b(t)$	$c(t) = a(t)\,b(t)$
frequency range	$(-f_a, f_a)$	$(-f_b, f_b)$	$(-f_a - f_b, f_a + f_b)$

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Aliasing rejection

(Global sol.) Oversample the input/Downsample the output: factor N for a VS truncated at order N.

(Local sol.) Idem with a factor 2 for each product of two signals.

In summary...

Derivation of the transfer kernels

1. Use the cancelling system and interconnection laws...
2. ... to transform the weakly nonlinear problem into a infinite sequence of solvable linear equations

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Possible generalization to Partial Differential Equations

Same principle (cf. Applications)

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2 Séries de Volterra : généralités

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